Relaxing Instrument Exclusion with Common Confounders

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Setup: Unobserved Common Confounding

Outcome $Y \in \mathcal{Y} \subseteq \mathbb{R}$, treatment $A \in \mathcal{A} \subseteq \mathbb{R}d_A$, instruments $Z \in \mathcal{Z} \subseteq \mathbb{R}d_Z$, proxies $W \in \mathcal{W} \subseteq \mathbb{R}d_W$, unobserved common confounders $U \in \mathcal{U} \subseteq \mathbb{R}d_U$. All results hold conditional on covariates. Notation:
$\Sigma_{XY} := \mathbb{E}[XY^\intercal] - \mathbb{E}[X]\,\mathbb{E}[Y]^\intercal$, $\Sigma_X = \Sigma_{XX}$, $\hat{W}_Z := \mathbb{E}_{\mathrm{L}}[W|Z] = Z\Sigma_Z^{-1}\Sigma_{ZW}$.

Assumption 1 (IV Common Confounding Model)**.** *1. SUTVA:* $Y = Y(A, Z)$.

2. Instruments

(a) Exclusion: $Y(a, z) = Y(a) \perp\!\!\!\perp Z \mid U$.

(b) Index sufficiency: For some $\tau \in L_2(Z)$*, where* $T := \tau(Z)$, $U \perp\!\!\!\perp Z \mid T$.

(c) Relevance (completeness): For any $g(A,T) \in L_2(A,T)$,

$$\mathbb{E}[g(A,T)|Z] = 0 \text{ only when } g(A,T) = 0. \quad (1)$$

3. Proxies

(a) Exclusion: $W \perp\!\!\!\perp Z \mid U$.

(b) Relevance (completeness): For any $g(U) \in L_2(U)$,

$$\mathbb{E}[g(U)|W] = 0 \text{ only when } g(U) = 0. \quad (2)$$

Linear Model Example

Equation	Exclusion	Relevance	
$Y = A\beta + W\upsilon_Y + U\gamma_Y + \varepsilon_Y,$	$\mathbb{E}[\varepsilon_Y Z] = \mathbf{0},$		(3)
$A = Z\zeta + W\upsilon_A + U\gamma_W + \varepsilon_A,$		$\operatorname{rank}\left(\mathbb{E}\left[A^T Z \mid \hat{W}_Z\right]\right) = d_A,$	(4)
$Z = U\gamma_Z + \varepsilon_Z,$		$\operatorname{rank}(\gamma_Z) = d_U < d_Z,$	(5)
$W = U\gamma_W + \varepsilon_W,$	$\mathbb{E}[\varepsilon_W^\intercal \varepsilon_Z] = \mathbf{0},$	$\operatorname{rank}(\gamma_W) = d_U \leq d_W.$	(6)

Idea: Identify a Valid Control from Observables

Quantity of interest: *Causal effect* of A on Y.	$J = \int Y(a)\pi(a)\mathrm{d}a$
A is endogenous (simultaneity, unobserved confounders).	$Y(a) \not\perp\!\!\!\perp A$
We want to use relevant instruments Z for A.	$A(z) \neq A$
Instruments NOT unconditionally excluded.	$Y(a) \not\perp\!\!\!\perp Z$
The unobserved *common confounders* U fully explain the association between Z and W.	$Z \perp\!\!\!\perp W \mid U$
Instruments would be excluded conditional on the common confounders U.	$Y(a) \perp\!\!\!\perp Z \mid U$

Lemma 1. *Assume* $W \perp\!\!\!\perp Z \mid U$ *(A1.3a), and for any* $g(U) \in L_2(U)$, $\mathbb{E}[g(U)|W] = 0$ *only when* $g(U) = 0$ *(A1.3b). Take any* $\tau \in L_2(Z)$*, where* $T := \tau(Z)$*, such that* $W \perp\!\!\!\perp Z \mid T$*. Then, also* $U \perp\!\!\!\perp Z \mid T$.

In words: If W and Z are independent conditional on $\tau(Z)$ (part of Z's variation), then so are U and Z conditional on $\tau(Z)$. Exclusion is restored. Identification ensues in with outcome model separability [Imbens and Newey, 2009] or first stage monotonicity [Newey and Powell, 2003]. Index sufficiency with fixed effects [Liu et al., 2021] similar in spirit.

Motivation/Application: Returns to Education

Data: National Longitudinal Survey of Youth 1997. $n =$ 1,983.

Y	Household net worth at 35: continuous variable, in USD
A	BA degree: 1 if individual i obtained a BA degree, 0 o.w.
Z	Pre-college test results: subject GPA, ASVAB percentile
W	Risky behaviour dummies: drinking, smoking, etc by age 17
U	Ability: Unmeasured intellectual capacity
ε_Y	Disturbance: Heterogeneous characteristics, and chance
X	Covariates: sex, college GPA, family variables, region, etc

Utility-maximiser chooses education with knowledge $\mathcal{I}$ about ε_Y and U:

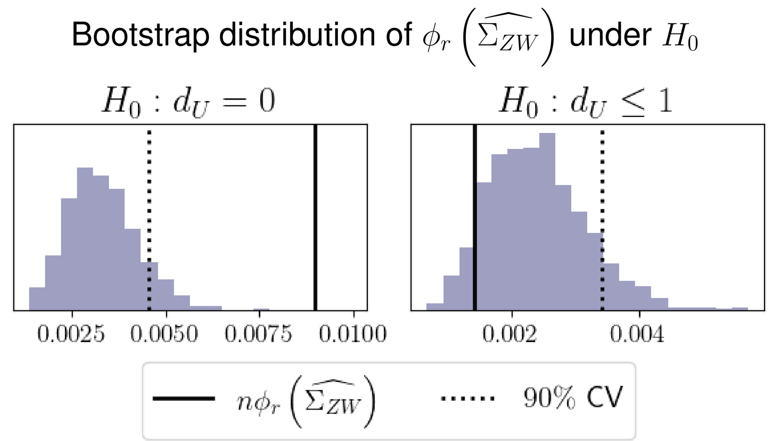
$$A = \underset{a \in \{0,1\}}{\arg\max} \left(\mathbb{E}\left[u(Y(a)) - c(a) | A = a, \mathcal{I}\right]\right), \quad (7)$$

Problem: Self-selection and unobserved confounding from ability ensue.

Test relevance of W and Z for U

For $r < \min\{d_Z, d_W\}$, use sum of r squared singular values $\phi_r(\Sigma_{ZW})$ to test

$$H_0 : d_U \leq r, \text{ vs } H_a : d_U > r. \quad (8)$$



Conclusion:

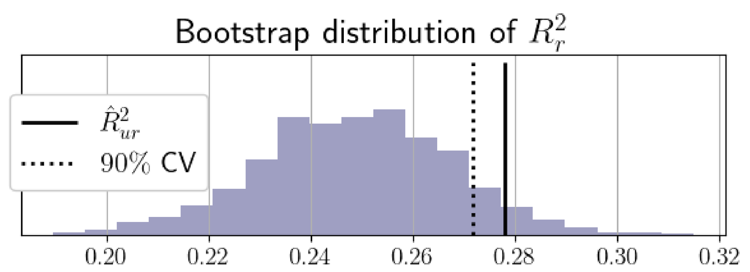
- Strong evidence for $d_U = 1$.
- W and Z relevant for U.

Test relevance of Z for A given T

Simple prediction test:

$H_0 : R^2_{ur} = R^2_r$, vs $R^2_{ur} > R^2_r$,

$$A = Z\tilde{\zeta} + X\tilde{\eta}_{A,ur} + \tilde{\varepsilon}_{A,ur} \implies R^2_{ur}$$
$$A = T\tilde{\gamma}_A + X\tilde{\eta}_{A,r} + \tilde{\varepsilon}_{A,r} \implies R^2_r$$



Conclusion: Z is relevant for A given T.

Exclusion of Z conditional on U

- Understand U through T to argue for/against exclusion (A1.2a).
- Here, T appears to hold a measure of general ability constant.
- Z probably excluded conditional on general ability and covariates.

Results

- Positive ability bias in OLS indicated by PL [Cui et al., 2020]
- Even larger negative selection bias in OLS indicated by IV
- ICC corrects for ability and selection bias
- About 50% larger SEs in ICC compared to IV

Estimates for β and linear parameter on T
(SE in parantheses)

	OLS	PL	IV	ICC
β	59.18 (9.12)	30.90 (10.40)	222.97 (34.74)	125.15 (52.93)
T		27.76 (4.82)		16.05 (7.37)

Intuition: Orthogonalisation wrt Unobservables

Create control function T

Decompose the covariance of Z and W as

$$\Sigma_{ZW} = \underbrace{\gamma_Z^\intercal \Sigma_U \gamma_W}_{\implies \operatorname{rank}(\Sigma_{ZW}) = d_U} := C_Z C_W^\intercal. \quad (9)$$

for some $C_Z \in \mathbb{R}^{d_Z \times d_U}$ and $C_W \in \mathbb{R}^{d_W \times d_U}$ s.t. a valid control function T is

$$T := Z\Sigma_Z^{-1}C_Z, \qquad \mathbb{E}_{\mathrm{L}}[W|Z] = TC_W^\intercal, \qquad \Sigma_{ZT} = C_Z, \Sigma_T = C_Z^\intercal \Sigma_Z^{-1} C_Z.$$

How does T help deconfound Z?

- $\mathbb{E}_{\mathrm{L}}[U|Z]$ is proportional to $\mathbb{E}_{\mathrm{L}}[W|Z]$, because $\operatorname{rank}(\gamma_W) = d_U$.
- By holding $\mathbb{E}_{\mathrm{L}}[W|Z]$ fixed via T, we are also holding $\mathbb{E}_{\mathrm{L}}[U|Z]$ fixed.
- All endogeneity in Z is from $\mathbb{E}_{\mathrm{L}}[U|Z]$, so fixed T restores exclusion.

E.g. for any $D_Z \in \mathbb{R}^{d_U \times d_A}$ s.t. $\operatorname{rank}\left(\left(I_{d_z} - \Sigma_T^{-1}\Sigma_{TZ}\right) D_Z\right) = d_A$ get deconfounded instrument $\tilde{Z}$ as

$$\tilde{Z} = Z \underbrace{\left(I_{d_z} - \Sigma_Z^{-1} C_Z \left(C_Z^\intercal \Sigma_Z^{-1} C_Z\right)^{-1} C_Z^\intercal\right)}_{:=M, \text{ note that } C_Z^\intercal M = \mathbf{0}} D_Z. \quad (10)$$

Consistent method of moments estimator

$$\beta = \Sigma_{\tilde{Z}A}^{-1}\Sigma_{\tilde{Z}Y} \implies \hat{\beta}_{MoM} = \left(D_Z^\intercal \hat{M}^\intercal Z^\intercal A\right)^{-1} \left(D_Z^\intercal \hat{M}^\intercal Z^\intercal Y\right) \quad (11)$$

Conclusion

- Using proxies to deconfound wrt common unobservables . . .
  - \+ can restore instrument exclusion when conditioning cannot, but
  - \- consumes more variation in the instruments than conditioning on observables, thus requiring rich relevance of instruments for treatment.
- Method most relevant with rich observational data, but intricate biases (like selection or simultaneity).

References

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